Mathematica 11.3 Integration Test Results

Test results for the 935 problems in "7.4.2 Exponentials of inverse hyperbolic cotangent functions.m"

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{2\,\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 14 leaves, 4 steps):

$$-\text{Log}[x]+2\,\text{Log}[1-a\,x]$$

Result (type 3, 29 leaves):

$$-\text{Log}\left[1-e^{2\,\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1+e^{2\,\text{ArcCoth}[a\,x]}\right]$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$-\text{Log}[x]+2\,\text{Log}[1+a\,x]$$

Result (type 3, 29 leaves):

$$-\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1+e^{-2\,\text{ArcCoth}[a\,x]}\right]$$

Problem 64: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2}\,\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 291 leaves, 17 steps):

$$-\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]+\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]+2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]+$$

$$2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 87 leaves):

$$2\,\text{ArcTan}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1-e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]+\text{Log}\left[1+e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]-$$

$$\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{-\text{ArcCoth}[a x]+2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]$$

Problem 65: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2}\text{ArcCoth}[a x]}}{x^2}\,dx$$

Optimal (type 3, 267 leaves, 13 steps):

$$a\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{1/4}-\frac{a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}+$$

$$\frac{a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2\sqrt{2}}-\frac{a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 70 leaves):

$$a\left(\frac{2\,e^{\frac{1}{2}\text{ArcCoth}[a x]}}{1+e^{2\,\text{ArcCoth}[a x]}}-\frac{1}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{-\text{ArcCoth}[a x]+2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

Problem 66: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2}\text{ArcCoth}[a x]}}{x^3}\,dx$$

Optimal (type 3, 319 leaves, 14 steps):

$$\frac{1}{4} a^2 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4}+\frac{1}{2} a^2 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{5/4}-\frac{a^2 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}}+$$
$$\frac{a^2 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}}+\frac{a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}-\frac{a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}$$

Result (type 7, 85 leaves):

$$\frac{1}{16} a^2 \left(\frac{8 e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(1+5 e^{2 \text{ArcCoth}[a x]}\right)}{\left(1+e^{2 \text{ArcCoth}[a x]}\right)^2}-\text{RootSum}\left[1+\#1^4\&,\frac{-\text{ArcCoth}[a x]+2 \text{Log}\left[e^{\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\&\right]\right)$$

Problem 67: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2} \text{ArcCoth}[a x]}}{x^4} \, dx$$

Optimal (type 3, 356 leaves, 15 steps):

$$\frac{3}{8} a^3 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4}+\frac{1}{12} a^3 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{5/4}+$$
$$\frac{a^2 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{5/4}}{3 x}-\frac{3 a^3 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+\frac{3 a^3 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+$$
$$\frac{3 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}-\frac{3 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}$$

Result (type 7, 93 leaves):

$$\frac{1}{96} a^3 \left(\frac{8 e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(9+6 e^{2 \text{ArcCoth}[a x]}+29 e^{4 \text{ArcCoth}[a x]}\right)}{\left(1+e^{2 \text{ArcCoth}[a x]}\right)^3}+9 \text{RootSum}\left[1+\#1^4\&,\frac{\text{ArcCoth}[a x]-2 \text{Log}\left[e^{\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\&\right]\right)$$

Problem 73: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2}\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 291 leaves, 17 steps):

$$-\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]+\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]-2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]+$$
$$2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 87 leaves):

$$-2\,\text{ArcTan}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1-e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right]+\text{Log}\left[1+e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-$$
$$\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{-\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1}\,\&\right]$$

Problem 74: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 268 leaves, 13 steps):

$$a\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}-\frac{3\,a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{3\,a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}-$$
$$\frac{3\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}+\frac{3\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 68 leaves):

$$a\left(\frac{2\,e^{\frac{3}{2}\text{ArcCoth}[a\,x]}}{1+e^{2\,\text{ArcCoth}[a\,x]}}+\frac{3}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a\,x]-2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1}\,\&\right]\right)$$

Problem 75: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2}\text{ArcCoth}[a\,x]}}{x^3}\,dx$$

Optimal (type 3, 319 leaves, 14 steps):

$$\frac{3}{4}a^2\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}+\frac{1}{2}a^2\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{7/4}-\frac{9\,a^2\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\sqrt{2}}+$$
$$\frac{9\,a^2\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\sqrt{2}}-\frac{9\,a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}+\frac{9\,a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}$$

Result (type 7, 84 leaves):

$$a^2\left(\frac{e^{\frac{3}{2}\text{ArcCoth}[a\,x]}\left(3+7\,e^{2\,\text{ArcCoth}[a\,x]}\right)}{2\left(1+e^{2\,\text{ArcCoth}[a\,x]}\right)^2}+\right.$$
$$\left.\frac{9}{16}\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a\,x]-2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1}\,\&\right]\right)$$

Problem 76: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2}\text{ArcCoth}[a\,x]}}{x^4}\,dx$$

Optimal (type 3, 356 leaves, 15 steps):

$$\frac{17}{24}a^3\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}+\frac{1}{4}a^3\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{7/4}+$$
$$\frac{a^2\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{7/4}}{3\,x}-\frac{17\,a^3\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}+\frac{17\,a^3\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}-$$
$$\frac{17\,a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{16\sqrt{2}}+\frac{17\,a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{16\sqrt{2}}$$

Result (type 7, 93 leaves):

$$\frac{1}{96} a^3 \left(\frac{8\, e^{\frac{3}{2}\text{ArcCoth}[a\,x]} \left(17 + 30\, e^{2\,\text{ArcCoth}[a\,x]} + 45\, e^{4\,\text{ArcCoth}[a\,x]}\right)}{\left(1 + e^{2\,\text{ArcCoth}[a\,x]}\right)^3} + 51\,\text{RootSum}\left[1 + \#1^4\,\&,\ \frac{\text{ArcCoth}[a\,x] - 2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]} - \#1\right]}{\#1}\,\&\right]\right)$$

Problem 82: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2}\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 320 leaves, 19 steps):

$$-\frac{8\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}} + \sqrt{2}\,\text{ArcTan}\left[1 - \frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right] - \sqrt{2}\,\text{ArcTan}\left[1 + \frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right] + 2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right] + 2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right] - \frac{\text{Log}\left[1 + \frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}} - \frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}} + \frac{\text{Log}\left[1 + \frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}} + \frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 97 leaves):

$$-8\,e^{\frac{1}{2}\text{ArcCoth}[a\,x]} + 2\,\text{ArcTan}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right] - \text{Log}\left[1 - e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right] + \text{Log}\left[1 + e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right] - \frac{1}{2}\,\text{RootSum}\left[1 + \#1^4\,\&,\ \frac{\text{ArcCoth}[a\,x] - 2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]} - \#1\right]}{\#1^3}\,\&\right]$$

Problem 83: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 299 leaves, 14 steps):

$$-5 a \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4}-\frac{4 a \left(1+\frac{1}{a x}\right)^{5/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}+\frac{5 a \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}-$$

$$\frac{5 a \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{5 a \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2 \sqrt{2}}+\frac{5 a \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2 \sqrt{2}}$$

Result (type 7, 80 leaves):

$$a \left(-8 e^{\frac{1}{2} \text{ArcCoth}[a x]}-\frac{2 e^{\frac{1}{2} \text{ArcCoth}[a x]}}{1+e^{2 \text{ArcCoth}[a x]}}-\frac{5}{4} \text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]-2 \text{Log}\left[e^{\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1^3} \&\right]\right)$$

Problem 84: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2} \text{ArcCoth}[a x]}}{x^3} \, dx$$

Optimal (type 3, 351 leaves, 15 steps):

$$-\frac{25}{4} a^2 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4}-\frac{5}{2} a^2 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{5/4}-$$

$$\frac{2 a^2 \left(1+\frac{1}{a x}\right)^{9/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}+\frac{25 a^2 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}}-\frac{25 a^2 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}}-$$

$$\frac{25 a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+\frac{25 a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}$$

Result (type 7, 94 leaves):

$$a^2 \left(-\frac{e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(25+45 e^{2 \text{ArcCoth}[a x]}+16 e^{4 \text{ArcCoth}[a x]}\right)}{2 \left(1+e^{2 \text{ArcCoth}[a x]}\right)^2}-\frac{25}{16} \text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]-2 \text{Log}\left[e^{\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1^3} \&\right]\right)$$

Problem 85: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^4}\,dx$$

Optimal (type 3, 385 leaves, 16 steps):

$$-\frac{55}{8}a^3\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{1/4}-\frac{11}{4}a^3\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{5/4}-\frac{2\,a^3\left(1+\frac{1}{a\,x}\right)^{9/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}-\frac{1}{3}a^3\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{9/4}+\frac{55\,a^3\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{55\,a^3\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{55\,a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{16\sqrt{2}}+\frac{55\,a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{16\sqrt{2}}$$

Result (type 7, 104 leaves):

$$a^3\left(-\left(\left(e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\left(165+462\,e^{2\,\text{ArcCoth}[a\,x]}+425\,e^{4\,\text{ArcCoth}[a\,x]}+96\,e^{6\,\text{ArcCoth}[a\,x]}\right)\right)\Big/\left(12\left(1+e^{2\,\text{ArcCoth}[a\,x]}\right)^3\right)\right)-\frac{55}{32}\text{RootSum}\left[1+\#1^4\&,\frac{\text{ArcCoth}[a\,x]-2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\&\right]\right)$$

Problem 91: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 291 leaves, 17 steps):

$$\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]-\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]-2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 85 leaves):

$$2\,\text{ArcTan}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1-e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]+\text{Log}\left[1+e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]$$

Problem 92: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 268 leaves, 13 steps):

$$-a\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}-\frac{a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}+\frac{a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 70 leaves):

$$a\left(-\frac{2\,e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}}{1+e^{-2\,\text{ArcCoth}[a\,x]}}-\frac{1}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{-\text{ArcCoth}[a\,x]-2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

Problem 93: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}}{x^3}\,dx$$

Optimal (type 3, 319 leaves, 14 steps):

$$\frac{1}{4}a^2\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}+\frac{1}{2}a^2\left(1-\frac{1}{a\,x}\right)^{5/4}\left(1+\frac{1}{a\,x}\right)^{3/4}+\frac{a^2\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\sqrt{2}}-\frac{a^2\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\sqrt{2}}+\frac{a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}$$

Result (type 7, 81 leaves):

$$\frac{1}{16} a^2 \left(\frac{8 e^{\frac{3}{2} \text{ArcCoth}[a x]} \left(5+e^{2 \text{ArcCoth}[a x]}\right)}{\left(1+e^{2 \text{ArcCoth}[a x]}\right)^2}-\text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]+2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1^3} \&\right]\right)$$

Problem 94: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2} \text{ArcCoth}[a x]}}{x^4} \, dx$$

Optimal (type 3, 356 leaves, 15 steps):

$$-\frac{3}{8} a^3 \left(1-\frac{1}{a x}\right)^{1/4} \left(1+\frac{1}{a x}\right)^{3/4}-\frac{1}{12} a^3 \left(1-\frac{1}{a x}\right)^{5/4} \left(1+\frac{1}{a x}\right)^{3/4}+$$
$$\frac{a^2 \left(1-\frac{1}{a x}\right)^{5/4} \left(1+\frac{1}{a x}\right)^{3/4}}{3 x}-\frac{3 a^3 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+\frac{3 a^3 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}-$$
$$\frac{3 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}+\frac{3 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}$$

Result (type 7, 93 leaves):

$$\frac{1}{96} a^3 \left(-\frac{8 e^{\frac{3}{2} \text{ArcCoth}[a x]} \left(29+6 e^{2 \text{ArcCoth}[a x]}+9 e^{4 \text{ArcCoth}[a x]}\right)}{\left(1+e^{2 \text{ArcCoth}[a x]}\right)^3}+\right.$$
$$\left.9 \text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]+2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1^3} \&\right]\right)$$

Problem 100: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2} \text{ArcCoth}[a x]}}{x} \, dx$$

Optimal (type 3, 291 leaves, 17 steps):

$$\sqrt{2}\ \text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]-\sqrt{2}\ \text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]+2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]+$$

$$2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 85 leaves):

$$-2\,\text{ArcTan}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1-e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]+\text{Log}\left[1+e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-$$

$$\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1}\,\&\right]$$

Problem 101: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 269 leaves, 13 steps):

$$-a\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{1/4}-\frac{3\,a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{3\,a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+$$

$$\frac{3\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}-\frac{3\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 68 leaves):

$$a\left(-\frac{2\,e^{-\frac{3}{2}\text{ArcCoth}[a\,x]}}{1+e^{-2\,\text{ArcCoth}[a\,x]}}+\frac{3}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1}\,\&\right]\right)$$

Problem 102: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2}\text{ArcCoth}[a\,x]}}{x^3}\,dx$$

Optimal (type 3, 319 leaves, 14 steps):

$$\frac{3}{4} a^2 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4}+\frac{1}{2} a^2 \left(1-\frac{1}{a x}\right)^{7/4} \left(1+\frac{1}{a x}\right)^{1/4}+\frac{9 a^2 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}}-$$

$$\frac{9 a^2 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}}-\frac{9 a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+\frac{9 a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}$$

Result (type 7, 84 leaves):

$$a^2 \left(\frac{e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(7+3 e^{2 \text{ArcCoth}[a x]}\right)}{2 \left(1+e^{2 \text{ArcCoth}[a x]}\right)^2}-\frac{9}{16} \text{RootSum}\left[1+\#1^4 \&,\frac{\text{ArcCoth}[a x]+2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1}\&\right]\right)$$

Problem 103: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2} \text{ArcCoth}[a x]}}{x^4} \, dx$$

Optimal (type 3, 356 leaves, 15 steps):

$$-\frac{17}{24} a^3 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4}-\frac{1}{4} a^3 \left(1-\frac{1}{a x}\right)^{7/4} \left(1+\frac{1}{a x}\right)^{1/4}+$$

$$\frac{a^2 \left(1-\frac{1}{a x}\right)^{7/4} \left(1+\frac{1}{a x}\right)^{1/4}}{3 x}-\frac{17 a^3 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+\frac{17 a^3 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+$$

$$\frac{17 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}-\frac{17 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}$$

Result (type 7, 93 leaves):

$$\frac{1}{96} a^3 \left(-\frac{8 e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(45+30 e^{2 \text{ArcCoth}[a x]}+17 e^{4 \text{ArcCoth}[a x]}\right)}{\left(1+e^{2 \text{ArcCoth}[a x]}\right)^3}+51 \text{RootSum}\left[1+\#1^4 \&,\frac{\text{ArcCoth}[a x]+2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1}\&\right]\right)$$

Problem 109: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 320 leaves, 19 steps):

$$-\frac{8\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}-\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]+\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]-2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 99 leaves):

$$-8\,e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}+2\,\text{ArcTan}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1-e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]+\text{Log}\left[1+e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{-\text{ArcCoth}[a\,x]-2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]$$

Problem 110: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 299 leaves, 14 steps):

$$\frac{4\,a\left(1-\frac{1}{a\,x}\right)^{5/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}+5\,a\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}+\frac{5\,a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{5\,a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{5\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}-\frac{5\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 80 leaves):

$$a\left(8\,e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}+\frac{2\,e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}}{1+e^{-2\,\text{ArcCoth}[a\,x]}}-\frac{5}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

Problem 111: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^3}\,dx$$

Optimal (type 3, 351 leaves, 15 steps):

$$-\frac{2\,a^2\left(1-\frac{1}{a\,x}\right)^{9/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}-\frac{25}{4}\,a^2\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}-\frac{5}{2}\,a^2\left(1-\frac{1}{a\,x}\right)^{5/4}\left(1+\frac{1}{a\,x}\right)^{3/4}-\frac{25\,a^2\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\,\sqrt{2}}+\frac{25\,a^2\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\,\sqrt{2}}-\frac{25\,a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\,\sqrt{2}}+\frac{25\,a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\,\sqrt{2}}$$

Result (type 7, 94 leaves):

$$a^2\left(-\frac{e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\left(16+45\,e^{2\,\text{ArcCoth}[a\,x]}+25\,e^{4\,\text{ArcCoth}[a\,x]}\right)}{2\left(1+e^{2\,\text{ArcCoth}[a\,x]}\right)^2}+\frac{25}{16}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

Problem 112: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^4}\,dx$$

Optimal (type 3, 385 leaves, 16 steps):

$$\frac{2 a^3 \left(1-\frac{1}{a x}\right)^{9/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{55}{8} a^3 \left(1-\frac{1}{a x}\right)^{1/4} \left(1+\frac{1}{a x}\right)^{3/4}+\frac{11}{4} a^3 \left(1-\frac{1}{a x}\right)^{5/4} \left(1+\frac{1}{a x}\right)^{3/4}+$$
$$\frac{1}{3} a^3 \left(1-\frac{1}{a x}\right)^{9/4} \left(1+\frac{1}{a x}\right)^{3/4}+\frac{55 a^3 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}-\frac{55 a^3 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+$$
$$\frac{55 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}-\frac{55 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}$$

Result (type 7, 104 leaves):

$$a^3 \left(\left(e^{-\frac{1}{2} \text{ArcCoth}[a x]} \left(96+425 e^{2 \text{ArcCoth}[a x]}+462 e^{4 \text{ArcCoth}[a x]}+165 e^{6 \text{ArcCoth}[a x]}\right)\right) \Big/ \left(12 \left(1+e^{2 \text{ArcCoth}[a x]}\right)^3\right)-\frac{55}{32} \text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]+2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1^3} \&\right]\right)$$

Problem 116: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{\text{ArcCoth}[x]}{3}}}{x} \, dx$$

Optimal (type 3, 402 leaves, 25 steps):

$$-\sqrt{3} \text{ArcTan}\left[\frac{1-\frac{2 \left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}}{\sqrt{3}}\right]+\sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 \left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}}{\sqrt{3}}\right]-\text{ArcTan}\left[\sqrt{3}-\frac{2 \left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+$$
$$\text{ArcTan}\left[\sqrt{3}+\frac{2 \left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+2 \text{ArcTan}\left[\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+2 \text{ArcTanh}\left[\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right]-$$
$$\frac{1}{2} \text{Log}\left[1+\frac{\left(1+\frac{1}{x}\right)^{1/3}}{\left(\frac{-1+x}{x}\right)^{1/3}}-\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right]+\frac{1}{2} \text{Log}\left[1+\frac{\left(1+\frac{1}{x}\right)^{1/3}}{\left(\frac{-1+x}{x}\right)^{1/3}}+\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right]+$$
$$\frac{1}{2} \sqrt{3} \text{Log}\left[1-\frac{\sqrt{3} \left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{1}{2} \sqrt{3} \text{Log}\left[1+\frac{\sqrt{3} \left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]$$

Result (type 7, 218 leaves):

$$-2\,\text{ArcTan}\left[e^{\frac{\text{ArcCoth}[x]}{3}}\right]+\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2\,e^{\frac{\text{ArcCoth}[x]}{3}}}{\sqrt{3}}\right]+\sqrt{3}\,\text{ArcTan}\left[\frac{1+2\,e^{\frac{\text{ArcCoth}[x]}{3}}}{\sqrt{3}}\right]-\text{Log}\left[1-e^{\frac{\text{ArcCoth}[x]}{3}}\right]+\text{Log}\left[1+e^{\frac{\text{ArcCoth}[x]}{3}}\right]-\frac{1}{2}\text{Log}\left[1-e^{\frac{\text{ArcCoth}[x]}{3}}+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]+\frac{1}{2}\text{Log}\left[1+e^{\frac{\text{ArcCoth}[x]}{3}}+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]+\frac{1}{3}\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\ \frac{1}{-\#1+2\,\#1^3}\left(2\,\text{ArcCoth}[x]-6\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]-\text{ArcCoth}[x]\,\#1^2+3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2\right)\&\right]$$

Problem 117: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{\text{ArcCoth}[x]}{3}}}{x^2}\,dx$$

Optimal (type 3, 233 leaves, 14 steps):

$$\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}-\frac{1}{3}\text{ArcTan}\left[\sqrt{3}-\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{1}{3}\text{ArcTan}\left[\sqrt{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{2}{3}\text{ArcTan}\left[\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{\text{Log}\left[1-\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{2\sqrt{3}}-\frac{\text{Log}\left[1+\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{2\sqrt{3}}$$

Result (type 7, 116 leaves):

$$\frac{2\,e^{\frac{\text{ArcCoth}[x]}{3}}}{1+e^{2\,\text{ArcCoth}[x]}}-\frac{2}{3}\text{ArcTan}\left[e^{\frac{\text{ArcCoth}[x]}{3}}\right]+\frac{1}{9}\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\ \frac{1}{-\#1+2\,\#1^3}\left(2\,\text{ArcCoth}[x]-6\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]-\text{ArcCoth}[x]\,\#1^2+3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2\right)\&\right]$$

Problem 118: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{\text{ArcCoth}[x]}{3}}}{x^3}\,dx$$

Optimal (type 3, 260 leaves, 15 steps):

$$\frac{1}{6}\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}-\frac{1}{18}\text{ArcTan}\left[\sqrt{3}-\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{1}{18}\text{ArcTan}\left[\sqrt{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{1}{9}\text{ArcTan}\left[\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{\text{Log}\left[1-\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{12\sqrt{3}}-\frac{\text{Log}\left[1+\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{12\sqrt{3}}$$

Result (type 7, 124 leaves):

$$\frac{1}{54}\left(\frac{18\,e^{\frac{\text{ArcCoth}[x]}{3}}\left(1+7\,e^{2\,\text{ArcCoth}[x]}\right)}{\left(1+e^{2\,\text{ArcCoth}[x]}\right)^2}-6\,\text{ArcTan}\left[e^{\frac{\text{ArcCoth}[x]}{3}}\right]+\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\ \frac{1}{-\#1+2\,\#1^3}\left(2\,\text{ArcCoth}[x]-6\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]-\text{ArcCoth}[x]\,\#1^2+3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2\right)\&\right]\right)$$

Problem 119: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{\text{ArcCoth}[x]}{3}}}{x^4}\,dx$$

Optimal (type 3, 287 leaves, 16 steps):

$$\frac{19}{54}\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3\,x}-\frac{19}{162}\text{ArcTan}\left[\sqrt{3}-\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{19}{162}\text{ArcTan}\left[\sqrt{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{19}{81}\text{ArcTan}\left[\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{19\,\text{Log}\left[1-\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{108\sqrt{3}}-\frac{19\,\text{Log}\left[1+\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{108\sqrt{3}}$$

Result (type 7, 133 leaves):

$$\frac{1}{486}\left(\frac{18\,e^{\frac{\text{ArcCoth}[x]}{3}}\left(19+8\,e^{2\,\text{ArcCoth}[x]}+61\,e^{4\,\text{ArcCoth}[x]}\right)}{\left(1+e^{2\,\text{ArcCoth}[x]}\right)^3}-114\,\text{ArcTan}\left[e^{\frac{\text{ArcCoth}[x]}{3}}\right]-19\,\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\ \frac{1}{-\#1+2\,\#1^3}\left(-2\,\text{ArcCoth}[x]+6\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]+\text{ArcCoth}[x]\,\#1^2-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2\right)\&\right]\right)$$

Problem 123: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{x}\,dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$-\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\left(\frac{-1+x}{x}\right)^{1/3}}{\sqrt{3}\left(1+\frac{1}{x}\right)^{1/3}}\right]-\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\left(\frac{-1+x}{x}\right)^{1/3}}{\sqrt{3}\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{3}{2}\,\text{Log}\left[\left(1+\frac{1}{x}\right)^{1/3}-\left(\frac{-1+x}{x}\right)^{1/3}\right]-\frac{3}{2}\,\text{Log}\left[1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{1}{2}\,\text{Log}\left[1+\frac{1}{x}\right]-\frac{\text{Log}[x]}{2}$$

Result (type 7, 217 leaves):

$$\frac{1}{6}\left(4\,\text{ArcCoth}[x]+3\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2\,e^{\frac{\text{ArcCoth}[x]}{3}}}{\sqrt{3}}\right]-2\sqrt{3}\,\text{ArcTan}\left[\frac{1+2\,e^{\frac{\text{ArcCoth}[x]}{3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[1-e^{\frac{\text{ArcCoth}[x]}{3}}\right]-2\,\text{Log}\left[1+e^{\frac{\text{ArcCoth}[x]}{3}}\right]-2\,\text{Log}\left[1+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]+\text{Log}\left[1-e^{\frac{\text{ArcCoth}[x]}{3}}+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]+\text{Log}\left[1+e^{\frac{\text{ArcCoth}[x]}{3}}+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]\right)+2\,\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\frac{1}{-2+\#1^2}\left(\text{ArcCoth}[x]-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]+\text{ArcCoth}[x]\,\#1^2-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2\right)\&\right]\right)$$

Problem 124: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{x^2}\,dx$$

Optimal (type 3, 99 leaves, 3 steps):

$$\left(1+\frac{1}{x}\right)^{1/3}\left(\frac{-1+x}{x}\right)^{2/3}-\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\left(\frac{-1+x}{x}\right)^{1/3}}{\sqrt{3}\left(1+\frac{1}{x}\right)^{1/3}}\right]}{\sqrt{3}}-\text{Log}\left[1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{1}{3}\,\text{Log}\left[1+\frac{1}{x}\right]$$

Result (type 7, 112 leaves):

$$\frac{2}{9}\left(\frac{9\,e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{1+e^{2\,\text{ArcCoth}[x]}}+2\,\text{ArcCoth}[x]-3\,\text{Log}\left[1+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]+\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\right.\right.$$
$$\left.\left.\frac{1}{-2+\#1^2}\left(\text{ArcCoth}[x]-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]+\text{ArcCoth}[x]\,\#1^2-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2\right)\&\right]\right)$$

Problem 125: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{x^3}\,dx$$

Optimal (type 3, 130 leaves, 4 steps):

$$\frac{1}{3}\left(1+\frac{1}{x}\right)^{1/3}\left(\frac{-1+x}{x}\right)^{2/3}+\frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}-$$
$$\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\left(\frac{-1+x}{x}\right)^{1/3}}{\sqrt{3}\left(1+\frac{1}{x}\right)^{1/3}}\right]}{3\sqrt{3}}-\frac{1}{3}\,\text{Log}\left[1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{1}{9}\,\text{Log}\left[1+\frac{1}{x}\right]$$

Result (type 7, 134 leaves):

$$-\frac{2}{27}$$
$$\left(\frac{27\,e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{\left(1+e^{2\,\text{ArcCoth}[x]}\right)^2}-\frac{36\,e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{1+e^{2\,\text{ArcCoth}[x]}}-2\,\text{ArcCoth}[x]+3\,\text{Log}\left[1+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]-\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\right.\right.$$
$$\left.\left.\frac{1}{-2+\#1^2}\left(\text{ArcCoth}[x]-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]+\text{ArcCoth}[x]\,\#1^2-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2\right)\&\right]\right)$$

Problem 126: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4}\text{ArcCoth}[a\,x]}\,x^2\,dx$$

Optimal (type 3, 429 leaves, 19 steps):

$$\frac{37\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x}{96 a^2}+\frac{3\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x^2}{8 a}+\frac{1}{3}\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x^3-$$
$$\frac{11 \operatorname{ArcTan}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{64 \sqrt{2} a^3}+\frac{11 \operatorname{ArcTan}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{64 \sqrt{2} a^3}+\frac{11 \operatorname{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{64 a^3}+$$
$$\frac{11 \operatorname{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{64 a^3}-\frac{11 \operatorname{Log}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{128 \sqrt{2} a^3}+\frac{11 \operatorname{Log}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{128 \sqrt{2} a^3}$$

Result (type 7, 167 leaves):

$$\frac{1}{1536 a^3}\left(-4\left(-\frac{1024 e^{\frac{1}{4} \operatorname{ArcCoth}[a x]}}{\left(-1+e^{2 \operatorname{ArcCoth}[a x]}\right)^3}-\frac{1600 e^{\frac{1}{4} \operatorname{ArcCoth}[a x]}}{\left(-1+e^{2 \operatorname{ArcCoth}[a x]}\right)^2}-\frac{840 e^{\frac{1}{4} \operatorname{ArcCoth}[a x]}}{-1+e^{2 \operatorname{ArcCoth}[a x]}}-66 \operatorname{ArcTan}\left[e^{\frac{1}{4} \operatorname{ArcCoth}[a x]}\right]+\right.\right.$$
$$\left.33 \operatorname{Log}\left[1-e^{\frac{1}{4} \operatorname{ArcCoth}[a x]}\right]-33 \operatorname{Log}\left[1+e^{\frac{1}{4} \operatorname{ArcCoth}[a x]}\right]\right)-$$
$$\left.33 \operatorname{RootSum}\left[1+\#1^4 \&, \frac{\operatorname{ArcCoth}[a x]-4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcCoth}[a x]}-\#1\right]}{\#1^3} \&\right]\right)$$

Problem 127: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} \operatorname{ArcCoth}[a x]} x \, dx$$

Optimal (type 3, 392 leaves, 17 steps):

$$\frac{\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x}{8 a}+\frac{1}{2}\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{9/8} x^2-$$
$$\frac{\operatorname{ArcTan}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{16 \sqrt{2} a^2}+\frac{\operatorname{ArcTan}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{16 \sqrt{2} a^2}+\frac{\operatorname{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{16 a^2}+$$
$$\frac{\operatorname{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{16 a^2}-\frac{\operatorname{Log}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{32 \sqrt{2} a^2}+\frac{\operatorname{Log}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{32 \sqrt{2} a^2}$$

Result (type 7, 141 leaves):

$$\frac{1}{128 a^2}\left(-4\left(-\frac{64 e^{\frac{1}{4}\text{ArcCoth}[a x]}}{\left(-1+e^{2 \text{ArcCoth}[a x]}\right)^2}-\frac{72 e^{\frac{1}{4}\text{ArcCoth}[a x]}}{-1+e^{2 \text{ArcCoth}[a x]}}-2 \text{ArcTan}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]+\text{Log}\left[1-e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1+e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]\right)-\text{RootSum}\left[1+\#1^4\&,\frac{\text{ArcCoth}[a x]-4 \text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\&\right]\right)$$

Problem 128: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4}\text{ArcCoth}[a x]}\,dx$$

Optimal (type 3, 352 leaves, 16 steps):

$$\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x-\frac{\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{2\sqrt{2}\, a}+\frac{\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{2\sqrt{2}\, a}+\frac{\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{2 a}+\frac{\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{2 a}-\frac{\text{Log}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{4\sqrt{2}\, a}+\frac{\text{Log}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{4\sqrt{2}\, a}$$

Result (type 7, 117 leaves):

$$\frac{1}{16 a}\left(-4\left(-\frac{8 e^{\frac{1}{4}\text{ArcCoth}[a x]}}{-1+e^{2 \text{ArcCoth}[a x]}}-2 \text{ArcTan}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]+\text{Log}\left[1-e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1+e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]\right)-\text{RootSum}\left[1+\#1^4\&,\frac{\text{ArcCoth}[a x]-4 \text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\&\right]\right)$$

Problem 129: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4}\text{ArcCoth}[a x]}}{x}\,dx$$

Optimal (type 3, 919 leaves, 39 steps):

$$-\sqrt{2+\sqrt{2}}\ \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-\frac{2\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]-\sqrt{2-\sqrt{2}}\ \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-\frac{2\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+$$

$$\sqrt{2+\sqrt{2}}\ \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+\frac{2\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]+\sqrt{2-\sqrt{2}}\ \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+\frac{2\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]-$$

$$\sqrt{2}\ \text{ArcTan}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a\,x}\right)^{1/8}}{\left(1-\frac{1}{a\,x}\right)^{1/8}}\right]+\sqrt{2}\ \text{ArcTan}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a\,x}\right)^{1/8}}{\left(1-\frac{1}{a\,x}\right)^{1/8}}\right]+2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/8}}{\left(1-\frac{1}{a\,x}\right)^{1/8}}\right]+$$

$$2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/8}}{\left(1-\frac{1}{a\,x}\right)^{1/8}}\right]+\frac{1}{2}\sqrt{2-\sqrt{2}}\ \text{Log}\left[1+\frac{\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}-\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}\right]-$$

$$\frac{1}{2}\sqrt{2-\sqrt{2}}\ \text{Log}\left[1+\frac{\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}+\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}\right]+$$

$$\frac{1}{2}\sqrt{2+\sqrt{2}}\ \text{Log}\left[1+\frac{\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}-\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}\right]-$$

$$\frac{1}{2}\sqrt{2+\sqrt{2}}\ \text{Log}\left[1+\frac{\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}+\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}\right]-$$

$$\frac{\text{Log}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a\,x}\right)^{1/8}}{\left(1-\frac{1}{a\,x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a\,x}\right)^{1/8}}{\left(1-\frac{1}{a\,x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 128 leaves):

$$2\,\text{ArcTan}\left[e^{\frac{1}{4}\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1-e^{\frac{1}{4}\text{ArcCoth}[a\,x]}\right]+\text{Log}\left[1+e^{\frac{1}{4}\text{ArcCoth}[a\,x]}\right]-$$

$$\frac{1}{4}\text{RootSum}\left[1+\#1^4\ \&,\ \frac{\text{ArcCoth}[a\,x]-4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\ \&\right]-$$

$$\frac{1}{4}\text{RootSum}\left[1+\#1^8\ \&,\ \frac{-\text{ArcCoth}[a\,x]+4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^7}\ \&\right]$$

Problem 130: Result is not expressed in closed-form.

$$\int\frac{e^{\frac{1}{4}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 676 leaves, 25 steps):

$$a\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8}-\frac{1}{4}\sqrt{2+\sqrt{2}}\ a\,\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]-$$
$$\frac{1}{4}\sqrt{2-\sqrt{2}}\ a\,\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+$$
$$\frac{1}{4}\sqrt{2+\sqrt{2}}\ a\,\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]+\frac{1}{4}\sqrt{2-\sqrt{2}}\ a\,\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+$$
$$\frac{1}{8}\sqrt{2-\sqrt{2}}\ a\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-$$
$$\frac{1}{8}\sqrt{2-\sqrt{2}}\ a\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]+$$
$$\frac{1}{8}\sqrt{2+\sqrt{2}}\ a\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-$$
$$\frac{1}{8}\sqrt{2+\sqrt{2}}\ a\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]$$

Result (type 7, 70 leaves):

$$a\left(\frac{2\,e^{\frac{1}{4}\text{ArcCoth}[a x]}}{1+e^{2\,\text{ArcCoth}[a x]}}-\frac{1}{16}\,\text{RootSum}\left[1+\#1^8\ \&,\ \frac{-\text{ArcCoth}[a x]+4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^7}\ \&\right]\right)$$

Problem 131: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4}\text{ArcCoth}[a x]}}{x^3}\,dx$$

Optimal (type 3, 731 leaves, 26 steps):

$$\frac{1}{8}a^2\left(1-\frac{1}{a\,x}\right)^{7/8}\left(1+\frac{1}{a\,x}\right)^{1/8}+\frac{1}{2}a^2\left(1-\frac{1}{a\,x}\right)^{7/8}\left(1+\frac{1}{a\,x}\right)^{9/8}-$$

$$\frac{1}{32}\sqrt{2+\sqrt{2}}\;a^2\,\mathrm{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-\frac{2\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]-\frac{1}{32}\sqrt{2-\sqrt{2}}\;a^2\,\mathrm{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-\frac{2\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+$$

$$\frac{1}{32}\sqrt{2+\sqrt{2}}\;a^2\,\mathrm{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+\frac{2\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]+\frac{1}{32}\sqrt{2-\sqrt{2}}\;a^2\,\mathrm{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+\frac{2\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+$$

$$\frac{1}{64}\sqrt{2-\sqrt{2}}\;a^2\,\mathrm{Log}\left[1+\frac{\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}-\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}\right]-$$

$$\frac{1}{64}\sqrt{2-\sqrt{2}}\;a^2\,\mathrm{Log}\left[1+\frac{\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}+\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}\right]+$$

$$\frac{1}{64}\sqrt{2+\sqrt{2}}\;a^2\,\mathrm{Log}\left[1+\frac{\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}-\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}\right]-$$

$$\frac{1}{64}\sqrt{2+\sqrt{2}}\;a^2\,\mathrm{Log}\left[1+\frac{\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}+\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a\,x}\right)^{1/8}}{\left(1+\frac{1}{a\,x}\right)^{1/8}}\right]$$

Result (type 7, 85 leaves) :

$$\frac{1}{128}a^2\left(\frac{32\,e^{\frac{1}{4}\mathrm{ArcCoth}[a\,x]}\left(1+9\,e^{2\,\mathrm{ArcCoth}[a\,x]}\right)}{\left(1+e^{2\,\mathrm{ArcCoth}[a\,x]}\right)^2}-\mathrm{RootSum}\left[1+\#1^8\,\&,\frac{-\mathrm{ArcCoth}[a\,x]+4\,\mathrm{Log}\left[e^{\frac{1}{4}\mathrm{ArcCoth}[a\,x]}-\#1\right]}{\#1^7}\,\&\right]\right)$$

Problem 133: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{3\,\mathrm{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 5, 151 leaves, 9 steps) :

$$-\frac{3\,x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}\left(-1-m\right),\frac{1-m}{2},\frac{1}{a^2\,x^2}\right]}{1+m}-\frac{x^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2\,x^2}\right]}{a\,m}+\frac{4\,x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{1}{2}\left(-1-m\right),\frac{1-m}{2},\frac{1}{a^2\,x^2}\right]}{1+m}+\frac{4\,x^m\,\text{Hypergeometric2F1}\left[\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2\,x^2}\right]}{a\,m}$$

Result (type 6, 381 leaves) :

$$\frac{1}{1+m}x^{1+m}\left(\left(\left(4\left(1+m\right)^2\sqrt{1-\frac{1}{a^2\,x^2}}\sqrt{\frac{1+a\,x}{a^2}}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{3}{2},1+m,-a\,x,a\,x\right]\right)\Big/\left(m\left(-1+a\,x\right)^{3/2}\sqrt{-\frac{1}{a^2}+x^2}\left(2\left(1+m\right)\text{AppellF1}\left[m,-\frac{1}{2},\frac{3}{2},1+m,-a\,x,a\,x\right]+a\,x\left(3\,\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{5}{2},2+m,-a\,x,a\,x\right]+\text{AppellF1}\left[1+m,\frac{1}{2},\frac{3}{2},2+m,-a\,x,a\,x\right]\right)\right)\right)+\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2\,x^2}\right]-\left(6\left(1+m\right)^2\sqrt{1-\frac{1}{a^2\,x^2}}\sqrt{1-a\,x}\sqrt{\frac{1+a\,x}{a^2}}\sqrt{1-a^2\,x^2}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,-a\,x,a\,x\right]\right)\Big/\left(m\left(-1+a\,x\right)^{3/2}\sqrt{1+a\,x}\sqrt{-\frac{1}{a^2}+x^2}\left(2\left(1+m\right)\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,-a\,x,a\,x\right]+a\,x\left(\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{3}{2},2+m,-a\,x,a\,x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},a^2\,x^2\right]\right)\right)\right)\right)$$

Problem 135: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 5, 74 leaves, 4 steps) :

$$\frac{x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}\left(-1-m\right),\frac{1-m}{2},\frac{1}{a^2\,x^2}\right]}{1+m}+\frac{x^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2\,x^2}\right]}{a\,m}$$

Result (type 6, 232 leaves) :

$$\frac{1}{1+m}x^{1+m}\left(\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right]-\right.$$
$$\left(2(1+m)^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1-ax}\sqrt{\frac{1+ax}{a^2}}\sqrt{1-a^2x^2}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right]\right)\Big/$$
$$\left(m(-1+ax)^{3/2}\sqrt{1+ax}\sqrt{-\frac{1}{a^2}+x^2}\right.$$
$$\left(2(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right]+ax\left(\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{3}{2},\right.\right.\right.$$
$$\left.\left.\left.\left.2+m,-ax,ax\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},a^2x^2\right]\right)\right)\right)\right)$$

Problem 136: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{-\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 5, 75 leaves, 4 steps):

$$\frac{x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right]}{1+m}-\frac{x^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right]}{a\,m}$$

Result (type 6, 199 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right]+\right.$$
$$\left(2(1+m)^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,ax,-ax\right]\right)\Big/$$
$$\left(m\sqrt{1+ax}\sqrt{-\frac{1}{a^2}+x^2}\left(-2(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,ax,-ax\right]+ax\left(\text{AppellF1}\left[1+m,\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.-\frac{1}{2},\frac{3}{2},2+m,ax,-ax\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},a^2x^2\right]\right)\right)\right)\right)$$

Problem 138: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{-3\,\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 5, 150 leaves, 9 steps):

$$-\frac{3\,x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right]}{1+m}+\frac{x^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right]}{a\,m}+\frac{4\,x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right]}{1+m}-\frac{4\,x^m\,\text{Hypergeometric2F1}\left[\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right]}{a\,m}$$

Result (type 6, 349 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\left(4(1+m)^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+a\,x}{a^2}}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{3}{2},1+m,a\,x,-a\,x\right]\right)\Big/\left(m\,(1+a\,x)^{3/2}\sqrt{-\frac{1}{a^2}+x^2}\left(2(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{3}{2},1+m,a\,x,-a\,x\right]-a\,x\left(3\,\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{5}{2},2+m,a\,x,-a\,x\right]+\text{AppellF1}\left[1+m,\frac{1}{2},\frac{3}{2},2+m,a\,x,-a\,x\right]\right)\right)\right)+\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right]+\left(6(1+m)^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+a\,x}{a^2}}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,a\,x,-a\,x\right]\right)\Big/\left(m\sqrt{1+a\,x}\sqrt{-\frac{1}{a^2}+x^2}\left(-2(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,a\,x,-a\,x\right]+a\,x\left(\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{3}{2},2+m,a\,x,-a\,x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},a^2x^2\right]\right)\right)\right)\right)$$

Problem 139: Unable to integrate problem.

$$\int e^{\frac{5}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{5}{4},-\frac{5}{4},-m,\frac{1}{a\,x},-\frac{1}{a\,x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{5}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Problem 140: Unable to integrate problem.

$$\int e^{\frac{3}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{3}{4},-\frac{3}{4},-m,\frac{1}{a\,x},-\frac{1}{a\,x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{3}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Problem 141: Unable to integrate problem.

$$\int e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{1}{4},-\frac{1}{4},-m,\frac{1}{a\,x},-\frac{1}{a\,x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Problem 142: Unable to integrate problem.

$$\int e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{a\,x},-\frac{1}{a\,x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Problem 143: Unable to integrate problem.

$$\int e^{-\frac{3}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,-\frac{3}{4},\frac{3}{4},-m,\frac{1}{a\,x},-\frac{1}{a\,x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{3}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Problem 144: Unable to integrate problem.

$$\int e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,-\frac{5}{4},\frac{5}{4},-m,\frac{1}{a\,x},-\frac{1}{a\,x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Problem 145: Unable to integrate problem.

$$\int e^{\frac{2\,\text{ArcCoth}[x]}{3}}\,x^m\,dx$$

Optimal (type 6, 34 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{1}{3},-\frac{1}{3},-m,\frac{1}{x},-\frac{1}{x}\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{2\,\text{ArcCoth}[x]}{3}}\,x^m\,dx$$

Problem 146: Unable to integrate problem.

$$\int e^{\frac{\text{ArcCoth}[x]}{3}}\,x^m\,dx$$

Optimal (type 6, 34 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{1}{6},-\frac{1}{6},-m,\frac{1}{x},-\frac{1}{x}\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{\text{ArcCoth}[x]}{3}} x^m \, dx$$

Problem 147: Unable to integrate problem.

$$\int e^{\frac{1}{4}\text{ArcCoth}[a x]} x^m \, dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[-1-m, \frac{1}{8}, -\frac{1}{8}, -m, \frac{1}{a x}, -\frac{1}{a x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{1}{4}\text{ArcCoth}[a x]} x^m \, dx$$

Problem 148: Unable to integrate problem.

$$\int e^{n \,\text{ArcCoth}[a x]} x^m \, dx$$

Optimal (type 6, 45 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[-1-m, \frac{n}{2}, -\frac{n}{2}, -m, \frac{1}{a x}, -\frac{1}{a x}\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{n \,\text{ArcCoth}[a x]} x^m \, dx$$

Problem 211: Unable to integrate problem.

$$\int \frac{e^{-2 \,\text{ArcCoth}[a x]}}{c - a c x} \, dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$-\frac{\text{Log}[1 + a x]}{a c}$$

Result (type 8, 20 leaves):

$$\int \frac{e^{-2 \,\text{ArcCoth}[a x]}}{c - a c x} \, dx$$

Problem 212: Unable to integrate problem.

$$\int \frac{e^{-2 \,\text{ArcCoth}[a x]}}{(c - a c x)^2} \, dx$$

Optimal (type 3, 12 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[a\,x]}{a\,c^2}$$

Result (type 8, 20 leaves):

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^2}\,dx$$

Problem 295: Unable to integrate problem.

$$\int e^{\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a\,c\,x}\,dx$$

Optimal (type 5, 65 leaves, 3 steps):

$$\frac{2\,x^{1+m}\,\sqrt{c-a\,c\,x}\,\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{a\,x}\right]}{(3+2\,m)\,\sqrt{1-\frac{1}{a\,x}}}$$

Result (type 8, 23 leaves):

$$\int e^{\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a\,c\,x}\,dx$$

Problem 335: Unable to integrate problem.

$$\int e^{-\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a\,c\,x}\,dx$$

Optimal (type 5, 131 leaves, 4 steps):

$$\frac{2\,\sqrt{1+\frac{1}{a\,x}}\,x^{1+m}\,\sqrt{c-a\,c\,x}}{(3+2\,m)\,\sqrt{1-\frac{1}{a\,x}}}-\frac{2\,(5+4\,m)\,x^m\,\sqrt{c-a\,c\,x}\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2}-m,-\frac{1}{a\,x}\right]}{a\,(1+2\,m)\,(3+2\,m)\,\sqrt{1-\frac{1}{a\,x}}}$$

Result (type 8, 25 leaves):

$$\int e^{-\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a\,c\,x}\,dx$$

Problem 359: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{2+\frac{n}{2}}\,dx$$

Optimal (type 3, 278 leaves, 6 steps):

$$-\frac{\left(56+14 n+n^2\right) \left(1-\frac{1}{a x}\right)^{-2-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} (c-a c x)^{\frac{4+n}{2}}}{a (4+n) (6+n)}+\frac{2 \left(56+14 n+n^2\right) \left(1-\frac{1}{a x}\right)^{-2-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} (c-a c x)^{\frac{4+n}{2}}}{a^2 (6+n) \left(8+6 n+n^2\right) x}+\frac{(8+n) \left(1-\frac{1}{a x}\right)^{-2-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x (c-a c x)^{\frac{4+n}{2}}}{6+n}-\frac{\left(a-\frac{1}{x}\right) \left(1-\frac{1}{a x}\right)^{-2-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x (c-a c x)^{\frac{4+n}{2}}}{a}$$

Result (type 8, 26 leaves):

$$\int e^{n \text{ArcCoth}[a x]} (c-a c x)^{2+\frac{n}{2}} dx$$

Problem 360: Unable to integrate problem.

$$\int e^{n \text{ArcCoth}[a x]} (c-a c x)^{1+\frac{n}{2}} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{2 (6+n) \left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} (c-a c x)^{\frac{2+n}{2}}}{a (2+n) (4+n)}+\frac{2 \left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x (c-a c x)^{\frac{2+n}{2}}}{4+n}$$

Result (type 8, 26 leaves):

$$\int e^{n \text{ArcCoth}[a x]} (c-a c x)^{1+\frac{n}{2}} dx$$

Problem 362: Unable to integrate problem.

$$\int e^{n \text{ArcCoth}[a x]} (c-a c x)^{-1+\frac{n}{2}} dx$$

Optimal (type 5, 80 leaves, 3 steps):

$$\frac{1}{n} 2 \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{n/2} x (c-a c x)^{\frac{1}{2} (-2+n)} \text{Hypergeometric2F1}\left[1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right) x}\right]$$

Result (type 8, 26 leaves):

$$\int e^{n \text{ArcCoth}[a x]} (c-a c x)^{-1+\frac{n}{2}} dx$$

Problem 363: Unable to integrate problem.

$$\int e^{n \text{ArcCoth}[a x]} (c-a c x)^{-2+\frac{n}{2}} dx$$

Optimal (type 5, 88 leaves, 3 steps):

$$-\frac{1}{2-n}2\left(1-\frac{1}{a x}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} x\,(c-a c x)^{\frac{1}{2}(-4+n)}\,\text{Hypergeometric2F1}\left[2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 26 leaves):

$$\int e^{n\,\text{ArcCoth}[a x]}\,(c-a c x)^{-2+\frac{n}{2}}\,dx$$

Problem 364: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a x]}\,(c-a c x)^{p}\,dx$$

Optimal (type 5, 104 leaves, 3 steps):

$$\frac{1}{1+p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x\,(c-a c x)^{p}\,\text{Hypergeometric2F1}\left[\frac{1}{2}(n-2p),-1-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 20 leaves):

$$\int e^{n\,\text{ArcCoth}[a x]}\,(c-a c x)^{p}\,dx$$

Problem 365: Result more than twice size of optimal antiderivative.

$$\int e^{n\,\text{ArcCoth}[a x]}\,(c-a c x)^{3}\,dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{1}{a(8-n)}32 c^3\left(1-\frac{1}{a x}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left[5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]$$

Result (type 5, 190 leaves):

$$-\frac{1}{24 a(2+n)}c^3 e^{n\,\text{ArcCoth}[a x]}\left(e^{2\,\text{ArcCoth}[a x]} n\left(-48+44 n-12 n^2+n^3\right)\text{Hypergeometric2F1}\left[1,1+\frac{n}{2},2+\frac{n}{2},e^{2\,\text{ArcCoth}[a x]}\right]+(2+n)\left(a n^3 x+n^2\left(-1-12 a x+a^2 x^2\right)+2 n\left(6+21 a x-6 a^2 x^2+a^3 x^3\right)+6\left(-7-4 a x+6 a^2 x^2-4 a^3 x^3+a^4 x^4\right)+\left(-48+44 n-12 n^2+n^3\right)\text{Hypergeometric2F1}\left[1,\frac{n}{2},1+\frac{n}{2},e^{2\,\text{ArcCoth}[a x]}\right]\right)\right)$$

Problem 372: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{5/2}\,dx$$

Optimal (type 5, 98 leaves, 3 steps):

$$\frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,(c-a\,c\,x)^{5/2}\,\text{Hypergeometric2F1}\left[-\frac{7}{2},\frac{1}{2}(-5+n),-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 22 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{5/2}\,dx$$

Problem 373: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{3/2}\,dx$$

Optimal (type 5, 98 leaves, 3 steps):

$$\frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,(c-a\,c\,x)^{3/2}\,\text{Hypergeometric2F1}\left[-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 22 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{3/2}\,dx$$

Problem 374: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,\sqrt{c-a\,c\,x}\,dx$$

Optimal (type 5, 98 leaves, 3 steps):

$$\frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,\sqrt{c-a\,c\,x}\,\text{Hypergeometric2F1}\left[-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 22 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,\sqrt{c-a\,c\,x}\,dx$$

Problem 375: Unable to integrate problem.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{\sqrt{c-a\,c\,x}}\,dx$$

Optimal (type 5, 96 leaves, 3 steps):

$$\frac{1}{\sqrt{c-a\,c\,x}}2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,\text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+n}{2},\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{\sqrt{c-a\,c\,x}}\,dx$$

Problem 376: Unable to integrate problem.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^{3/2}}\,dx$$

Optimal (type 5, 96 leaves, 3 steps):

$$-\frac{1}{(c-a\,c\,x)^{3/2}}2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^{3/2}}\,dx$$

Problem 377: Unable to integrate problem.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^{5/2}}\,dx$$

Optimal (type 5, 167 leaves, 4 steps):

$$-\frac{a\left(1-\frac{1}{a\,x}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x^2}{(3+n)\,(c-a\,c\,x)^{5/2}}+$$

$$\left(a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{a\,x}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]\right)\Big/$$

$$\left((3+n)\,(c-a\,c\,x)^{5/2}\right)$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^{5/2}}\,dx$$

Problem 378: Unable to integrate problem.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^{7/2}}\,dx$$

Optimal (type 5, 245 leaves, 5 steps):

$$-\frac{a\left(1-\frac{1}{a\,x}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x^2}{(5+n)\,(c-a\,c\,x)^{7/2}}+\frac{3\,a^2\left(1-\frac{1}{a\,x}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8\,n+n^2\right)(c-a\,c\,x)^{7/2}}-$$
$$\left(3\,a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{a\,x}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x^3\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]\right)\Big/$$
$$\left(2\left(15+8\,n+n^2\right)(c-a\,c\,x)^{7/2}\right)$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^{7/2}}\,dx$$

Problem 426: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{\left(c-\frac{c}{a\,x}\right)^2}\,dx$$

Optimal (type 3, 18 leaves, 6 steps):

$$\frac{x}{c^2}-\frac{\text{ArcTanh}[a\,x]}{a\,c^2}$$

Result (type 3, 39 leaves):

$$\frac{x}{c^2}+\frac{\text{Log}[1-a\,x]}{2\,a\,c^2}-\frac{\text{Log}[1+a\,x]}{2\,a\,c^2}$$

Problem 545: Attempted integration timed out after 120 seconds.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^{3/2}dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$-\left(\left(2^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{a\,x}\right)^{3/2}\text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2}\left(-3+n\right),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2\,a},1+\frac{1}{a\,x}\right]\right)\Big/\left(a\,(2+n)\left(1-\frac{1}{a\,x}\right)^{3/2}\right)\right)$$

Result (type 1, 1 leaves):

???

Problem 546: Attempted integration timed out after 120 seconds.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\sqrt{c-\frac{c}{a\,x}}\,dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$-\left(\left(2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{a\,x}}\,\text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2}\left(-1+n\right),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2\,a},1+\frac{1}{a\,x}\right]\right)\Big/\left(a\,(2+n)\sqrt{1-\frac{1}{a\,x}}\right)\right)$$

Result (type 1, 1 leaves):

???

Problem 547: Attempted integration timed out after 120 seconds.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{a\,x}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}\text{AppellF1}\left[\frac{2+n}{2},\frac{1+n}{2},2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2\,a},1+\frac{1}{a\,x}\right]}{a\,(2+n)\sqrt{c-\frac{c}{a\,x}}}$$

Result (type 1, 1 leaves):

???

Problem 548: Attempted integration timed out after 120 seconds.

$$\int \frac{e^{n \text{ArcCoth}[a x]}}{\left(c-\frac{c}{a x}\right)^{3/2}} \, dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$-\frac{2^{-\frac{1}{2}-\frac{n}{2}} \left(1-\frac{1}{a x}\right)^{3/2} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} \text{AppellF1}\left[\frac{2+n}{2}, \frac{3+n}{2}, 2, \frac{4+n}{2}, \frac{a+\frac{1}{x}}{2 a}, 1+\frac{1}{a x}\right]}{a (2+n) \left(c-\frac{c}{a x}\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

Problem 549: Unable to integrate problem.

$$\int e^{n \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 6, 110 leaves, 3 steps):

$$-\frac{1}{a (2+n)} 2^{1-\frac{n}{2}+p} \left(1-\frac{1}{a x}\right)^{-p} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} \left(c-\frac{c}{a x}\right)^p \text{AppellF1}\left[\frac{2+n}{2}, \frac{1}{2} (n-2 p), 2, \frac{4+n}{2}, \frac{a+\frac{1}{x}}{2 a}, 1+\frac{1}{a x}\right]$$

Result (type 8, 24 leaves):

$$\int e^{n \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Problem 550: Unable to integrate problem.

$$\int e^{2 p \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 5, 67 leaves, 3 steps):

$$-\frac{1}{a (1+p)} \left(1-\frac{1}{a x}\right)^{-p} \left(1+\frac{1}{a x}\right)^{1+p} \left(c-\frac{c}{a x}\right)^p \text{Hypergeometric2F1}\left[2, 1+p, 2+p, 1+\frac{1}{a x}\right]$$

Result (type 8, 25 leaves):

$$\int e^{2 p \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Problem 551: Unable to integrate problem.

$$\int e^{-2 p \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 6, 93 leaves, 3 steps):

$$-\frac{1}{a\,(1-p)}4^p\left(1-\frac{1}{a\,x}\right)^{-p}\left(1+\frac{1}{a\,x}\right)^{1-p}\left(c-\frac{c}{a\,x}\right)^p \text{AppellF1}\left[1-p,\,-2\,p,\,2,\,2-p,\,\frac{a+\frac{1}{x}}{2\,a},\,1+\frac{1}{a\,x}\right]$$

Result (type 8, 25 leaves):

$$\int e^{-2\,p\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Problem 552: Unable to integrate problem.

$$\int e^{2\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Optimal (type 5, 57 leaves, 7 steps):

$$\left(c-\frac{c}{a\,x}\right)^p x+\frac{(2-p)\left(c-\frac{c}{a\,x}\right)^p \text{Hypergeometric2F1}\left[1,\,p,\,1+p,\,1-\frac{1}{a\,x}\right]}{a\,p}$$

Result (type 8, 24 leaves):

$$\int e^{2\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Problem 553: Unable to integrate problem.

$$\int e^{\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$-\frac{1}{3\,a}2^{\frac{1}{2}+p}\left(1-\frac{1}{a\,x}\right)^{-p}\left(1+\frac{1}{a\,x}\right)^{3/2}\left(c-\frac{c}{a\,x}\right)^p \text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2}-p,\,2,\,\frac{5}{2},\,\frac{a+\frac{1}{x}}{2\,a},\,1+\frac{1}{a\,x}\right]$$

Result (type 8, 22 leaves):

$$\int e^{\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Problem 554: Unable to integrate problem.

$$\int e^{-\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$-\frac{1}{a}2^{\frac{3}{2}+p}\left(1-\frac{1}{a\,x}\right)^{-p}\sqrt{1+\frac{1}{a\,x}}\left(c-\frac{c}{a\,x}\right)^p \text{AppellF1}\left[\frac{1}{2},\,-\frac{1}{2}-p,\,2,\,\frac{3}{2},\,\frac{a+\frac{1}{x}}{2\,a},\,1+\frac{1}{a\,x}\right]$$

Result (type 8, 24 leaves):

$$\int e^{-\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Problem 555: Unable to integrate problem.

$$\int e^{-2\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Optimal (type 5, 114 leaves, 9 steps):

$$\frac{\left(c-\frac{c}{a\,x}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{a\,x}\right)^{2+p}\text{Hypergeometric2F1}\left[1,\,2+p,\,3+p,\,\frac{a-\frac{1}{x}}{2\,a}\right]}{2\,a\,c^2\,(2+p)}-\frac{\left(c-\frac{c}{a\,x}\right)^{2+p}\text{Hypergeometric2F1}\left[1,\,2+p,\,3+p,\,1-\frac{1}{a\,x}\right]}{a\,c^2}$$

Result (type 8, 24 leaves):

$$\int e^{-2\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^p dx$$

Problem 569: Result unnecessarily involves higher level functions.

$$\int \frac{e^{2\,\text{ArcCoth}[a\,x]}}{c-a^2\,c\,x^2}\,dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$-\frac{1}{a\,c\,(1-a\,x)}$$

Result (type 3, 18 leaves):

$$\frac{e^{2\,\text{ArcCoth}[a\,x]}}{2\,a\,c}$$

Problem 584: Result more than twice size of optimal antiderivative.

$$\int e^{4\,\text{ArcCoth}[a\,x]}\left(c-a^2\,c\,x^2\right)^2 dx$$

Optimal (type 1, 17 leaves, 3 steps):

$$\frac{c^2\,(1+a\,x)^5}{5\,a}$$

Result (type 1, 49 leaves):

$$c^2\,x+2\,a\,c^2\,x^2+2\,a^2\,c^2\,x^3+a^3\,c^2\,x^4+\frac{1}{5}\,a^4\,c^2\,x^5$$

Problem 586: Result unnecessarily involves higher level functions.

$$\int \frac{e^{4\,\text{ArcCoth}[a\,x]}}{c-a^2\,c\,x^2}\,dx$$

Optimal (type 1, 13 leaves, 3 steps):

$$\frac{x}{c\,(1-a\,x)^2}$$

Result (type 3, 18 leaves):

$$\frac{e^{4\,\text{ArcCoth}[a\,x]}}{4\,a\,c}$$

Problem 602: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{c-a^2\,c\,x^2}\,dx$$

Optimal (type 1, 14 leaves, 3 steps):

$$\frac{1}{a\,c\,(1+a\,x)}$$

Result (type 3, 18 leaves):

$$-\frac{e^{-2\,\text{ArcCoth}[a\,x]}}{2\,a\,c}$$

Problem 647: Unable to integrate problem.

$$\int \frac{e^{-\text{ArcCoth}[a\,x]}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Optimal (type 3, 37 leaves, 3 steps):

$$\frac{\sqrt{1-\frac{1}{a^2\,x^2}}\;x\,\text{Log}[1+a\,x]}{\sqrt{c-a^2\,c\,x^2}}$$

Result (type 8, 26 leaves):

$$\int \frac{e^{-\text{ArcCoth}[a\,x]}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Problem 730: Unable to integrate problem.

$$\int e^{3\,\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{3\,x^m\,\sqrt{c-a^2\,c\,x^2}}{a\,(1+m)\,\sqrt{1-\frac{1}{a^2\,x^2}}}+\frac{x^{1+m}\,\sqrt{c-a^2\,c\,x^2}}{(2+m)\,\sqrt{1-\frac{1}{a^2\,x^2}}}-\frac{4\,x^m\,\sqrt{c-a^2\,c\,x^2}\;\text{Hypergeometric2F1}[1,\,1+m,\,2+m,\,a\,x]}{a\,(1+m)\,\sqrt{1-\frac{1}{a^2\,x^2}}}$$

Result (type 8, 29 leaves):

$$\int e^{3\,\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Problem 731: Result unnecessarily involves higher level functions.

$$\int e^{2\,\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Optimal (type 5, 172 leaves, 8 steps):

$$\frac{x^{1+m}\,\sqrt{c-a^2\,c\,x^2}}{2+m}-\frac{c\,(3+2\,m)\,x^{1+m}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2\,x^2\right]}{(1+m)\,(2+m)\,\sqrt{c-a^2\,c\,x^2}}-\frac{2\,a\,c\,x^{2+m}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2\,x^2\right]}{(2+m)\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 6, 192 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\frac{\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2\,x^2\right]}{\sqrt{1-a^2\,x^2}}+\left(4\,(2+m)\,\sqrt{-c\,(1+a\,x)}\,\text{AppellF1}\left[1+m,\frac{1}{2},-\frac{1}{2},2+m,a\,x,-a\,x\right]\right)\Big/\left(\sqrt{-1+a\,x}\,\left(2\,(2+m)\,\text{AppellF1}\left[1+m,\frac{1}{2},-\frac{1}{2},2+m,a\,x,-a\,x\right]+a\,x\,\left(\text{AppellF1}\left[2+m,\frac{3}{2},-\frac{1}{2},3+m,a\,x,-a\,x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},1+\frac{m}{2}\right\},\left\{2+\frac{m}{2}\right\},a^2\,x^2\right]\right)\right)\right)\right)$$

Problem 734: Result unnecessarily involves higher level functions.

$$\int e^{-2\,\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Optimal (type 5, 172 leaves, 8 steps):

$$\frac{x^{1+m}\,\sqrt{c-a^2\,c\,x^2}}{2+m}-\frac{c\,(3+2\,m)\,x^{1+m}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2\,x^2\right]}{(1+m)\,(2+m)\,\sqrt{c-a^2\,c\,x^2}}+\frac{2\,a\,c\,x^{2+m}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2\,x^2\right]}{(2+m)\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 6, 191 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\frac{\sqrt{c-a^2 c x^2}\ \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2 x^2\right]}{\sqrt{1-a^2 x^2}}+\left(4\ (2+m)\ \sqrt{c-a c x}\ \text{AppellF1}\left[1+m,\frac{1}{2},-\frac{1}{2},2+m,-a x,a x\right]\right)\Big/\left(\sqrt{1+a x}\ \left(-2\ (2+m)\ \text{AppellF1}\left[1+m,\frac{1}{2},-\frac{1}{2},2+m,-a x,a x\right]+a x\left(\text{AppellF1}\left[2+m,\frac{3}{2},-\frac{1}{2},3+m,-a x,a x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},1+\frac{m}{2}\right\},\left\{2+\frac{m}{2}\right\},a^2 x^2\right]\right)\right)\right)\right)$$

Problem 735: Unable to integrate problem.

$$\int e^{-3\,\text{ArcCoth}[a x]}\ x^m \sqrt{c-a^2 c x^2}\ dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$-\frac{3 x^m \sqrt{c-a^2 c x^2}}{a\ (1+m)\ \sqrt{1-\frac{1}{a^2 x^2}}}+\frac{x^{1+m} \sqrt{c-a^2 c x^2}}{(2+m)\ \sqrt{1-\frac{1}{a^2 x^2}}}+\frac{4 x^m \sqrt{c-a^2 c x^2}\ \text{Hypergeometric2F1}[1,1+m,2+m,-a x]}{a\ (1+m)\ \sqrt{1-\frac{1}{a^2 x^2}}}$$

Result (type 8, 29 leaves):

$$\int e^{-3\,\text{ArcCoth}[a x]}\ x^m \sqrt{c-a^2 c x^2}\ dx$$

Problem 736: Result more than twice size of optimal antiderivative.

$$\int e^{n\,\text{ArcCoth}[a x]}\ \left(c-a^2 c x^2\right)^3 dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{1}{a\ (8-n)}256\, c^3 \left(1-\frac{1}{a x}\right)^{4-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-8+n)}\ \text{Hypergeometric2F1}\left[8,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]$$

Result (type 5, 267 leaves):

$$-\frac{1}{5040\,a} c^3 e^{n\,\text{ArcCoth}[a\,x]} \Big(-912\,n + 58\,n^3 - n^5 - 5040\,a\,x + 912\,a\,n^2\,x - 58\,a\,n^4\,x + a\,n^6\,x + 1368\,a^2\,n\,x^2 - 64\,a^2\,n^3\,x^2 + a^2\,n^5\,x^2 + 5040\,a^3\,x^3 - 152\,a^3\,n^2\,x^3 + 2\,a^3\,n^4\,x^3 - 576\,a^4\,n\,x^4 + 6\,a^4\,n^3\,x^4 - 3024\,a^5\,x^5 + 24\,a^5\,n^2\,x^5 + 120\,a^6\,n\,x^6 + 720\,a^7\,x^7 + e^{2\,\text{ArcCoth}[a\,x]}\,n\,\left(-1152 + 576\,n + 104\,n^2 - 52\,n^3 - 2\,n^4 + n^5\right) \text{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, e^{2\,\text{ArcCoth}[a\,x]}\right] + \left(-2304 + 784\,n^2 - 56\,n^4 + n^6\right) \text{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, e^{2\,\text{ArcCoth}[a\,x]}\right]\Big)$$

Problem 737: Result more than twice size of optimal antiderivative.

$$\int e^{n\,\text{ArcCoth}[a\,x]} \left(c - a^2\,c\,x^2\right)^2 dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$\frac{1}{a\,(6-n)} 64\,c^2 \left(1 - \frac{1}{a\,x}\right)^{3-\frac{n}{2}} \left(1 + \frac{1}{a\,x}\right)^{\frac{1}{2}(-6+n)} \text{Hypergeometric2F1}\left[6, 3 - \frac{n}{2}, 4 - \frac{n}{2}, \frac{a - \frac{1}{x}}{a + \frac{1}{x}}\right]$$

Result (type 5, 179 leaves):

$$\frac{1}{120\,a} c^2\,e^{n\,\text{ArcCoth}[a\,x]} \Big(22\,n - n^3 + 120\,a\,x - 22\,a\,n^2\,x + a\,n^4\,x - 28\,a^2\,n\,x^2 + a^2\,n^3\,x^2 - 80\,a^3\,x^3 + 2\,a^3\,n^2\,x^3 + 6\,a^4\,n\,x^4 + 24\,a^5\,x^5 + e^{2\,\text{ArcCoth}[a\,x]}\,n\,\left(32 - 16\,n - 2\,n^2 + n^3\right) \text{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, e^{2\,\text{ArcCoth}[a\,x]}\right] + \left(64 - 20\,n^2 + n^4\right) \text{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, e^{2\,\text{ArcCoth}[a\,x]}\right]\Big)$$

Problem 744: Result more than twice size of optimal antiderivative.

$$\int e^{n\,\text{ArcCoth}[a\,x]} \left(c - a^2\,c\,x^2\right)^{3/2} dx$$

Optimal (type 5, 116 leaves, 3 steps):

$$\left(32 \left(1 - \frac{1}{a\,x}\right)^{\frac{5-n}{2}} \left(1 + \frac{1}{a\,x}\right)^{\frac{1}{2}(-5+n)} \left(c - a^2\,c\,x^2\right)^{3/2} \text{Hypergeometric2F1}\left[5, \frac{5-n}{2}, \frac{7-n}{2}, \frac{a - \frac{1}{x}}{a + \frac{1}{x}}\right]\right) \Big/ \left(a^4\,(5-n) \left(1 - \frac{1}{a^2\,x^2}\right)^{3/2} x^3\right)$$

Result (type 5, 280 leaves):

$$\frac{1}{192\,a\,\left(c-a^2\,c\,x^2\right)^{3/2}}\,c^2\,\left(96\,a^3\,c\,\left(1-\frac{1}{a^2\,x^2}\right)^{3/2}\,x^3\,\left(a\,e^{n\,\text{ArcCoth}[a\,x]}\,\sqrt{1-\frac{1}{a^2\,x^2}}\,x\,(n+a\,x)+2\,e^{(1+n)\,\text{ArcCoth}[a\,x]}\,(-1+n)\,\text{Hypergeometric2F1}\left[1,\frac{1+n}{2},\frac{3+n}{2},e^{2\,\text{ArcCoth}[a\,x]}\right]\right)-c\,\left(-1+a^2\,x^2\right)\left(2\,e^{n\,\text{ArcCoth}[a\,x]}\,\left(-1+a^2\,x^2\right)^2\,\left(-a\,\left(-21+n^2\right)\,x+2\,n\,\left(1-n^2+\left(3+n^2\right)\,\text{Cosh}[2\,\text{ArcCoth}[a\,x]]\right)+a\,\left(3+n^2\right)\,\sqrt{1-\frac{1}{a^2\,x^2}}\,x\,\text{Cosh}[3\,\text{ArcCoth}[a\,x]]\right)+16\,a\,e^{(1+n)\,\text{ArcCoth}[a\,x]}\,\left(-3+3\,n-n^2+n^3\right)\,\sqrt{1-\frac{1}{a^2\,x^2}}\,x\,\text{Hypergeometric2F1}\left[1,\frac{1+n}{2},\frac{3+n}{2},e^{2\,\text{ArcCoth}[a\,x]}\right]\right)\right)$$

Problem 762: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Optimal (type 5, 127 leaves, 3 steps):

$$\frac{1}{1+2\,p}\left(1-\frac{1}{a^2\,x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2\,p)}\left(1-\frac{1}{a\,x}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{a\,x}\right)^{1+\frac{n}{2}+p}x\,\left(c-a^2\,c\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-1-2\,p,\frac{1}{2}\,(n-2\,p),-2\,p,\frac{2}{\left(a+\frac{1}{x}\right)\,x}\right]$$

Result (type 8, 24 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Problem 765: Result more than twice size of optimal antiderivative.

$$\int e^{4\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Optimal (type 5, 63 leaves, 4 steps):

$$\frac{1}{a\,(1-p)}2^{2+p}\,c\,(1+a\,x)^{1-p}\,\left(c-a^2\,c\,x^2\right)^{-1+p}\,\text{Hypergeometric2F1}\left[-2-p,-1+p,p,\frac{1}{2}\,(1-a\,x)\right]$$

Result (type 5, 159 leaves):

$$\frac{1}{a\,(1+p)}\left(-(-1+a\,x)^2\right)^{-p}\,(-2+2\,a\,x)^p\,\left(1-a^2\,x^2\right)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\left(a\,(1+p)\,x\,\left(\frac{1}{2}-\frac{a\,x}{2}\right)^p\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,a^2\,x^2\right]-(1+a\,x)\,\left(1-a^2\,x^2\right)^p\,\left(2\,\text{Hypergeometric2F1}\left[1-p,\,1+p,\,2+p,\,\frac{1}{2}\,(1+a\,x)\right]-\text{Hypergeometric2F1}\left[2-p,\,1+p,\,2+p,\,\frac{1}{2}\,(1+a\,x)\right]\right)\right)$$

Problem 767: Result more than twice size of optimal antiderivative.

$$\int e^{2\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Optimal (type 5, 54 leaves, 4 steps):

$$\frac{1}{a\,p}2^{1+p}\,(1+a\,x)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-1-p,\,p,\,1+p,\,\frac{1}{2}\,(1-a\,x)\right]$$

Result (type 5, 133 leaves):

$$\frac{1}{a\,(1+p)}\left(-(-1+a\,x)^2\right)^{-p}\,(-2+2\,a\,x)^p\,\left(1-a^2\,x^2\right)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\left(a\,(1+p)\,x\,\left(\frac{1}{2}-\frac{a\,x}{2}\right)^p\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,a^2\,x^2\right]-(1+a\,x)\,\left(1-a^2\,x^2\right)^p\,\text{Hypergeometric2F1}\left[1-p,\,1+p,\,2+p,\,\frac{1}{2}\,(1+a\,x)\right]\right)$$

Problem 770: Result more than twice size of optimal antiderivative.

$$\int e^{-2\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Optimal (type 5, 55 leaves, 4 steps):

$$-\frac{1}{a\,p}2^{1+p}\,(1-a\,x)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-1-p,\,p,\,1+p,\,\frac{1}{2}\,(1+a\,x)\right]$$

Result (type 5, 125 leaves):

$$\frac{1}{a\,(1+p)}2^p\,(1+a\,x)^{-p}\,\left(1-a^2\,x^2\right)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\left(a\,(1+p)\,x\,\left(\frac{1}{2}+\frac{a\,x}{2}\right)^p\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,a^2\,x^2\right]-(-1+a\,x)\,\left(1-a^2\,x^2\right)^p\,\text{Hypergeometric2F1}\left[1-p,\,1+p,\,2+p,\,\frac{1}{2}-\frac{a\,x}{2}\right]\right)$$

Problem 933: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

Optimal (type 6, 116 leaves, 3 steps):

$$-\frac{1}{a\,(2+n+2\,p)}2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2\,x^2}\right)^{-p}\left(c-\frac{c}{a^2\,x^2}\right)^{p}\left(1+\frac{1}{a\,x}\right)^{1+\frac{n}{2}+p}\text{AppellF1}\left[1+\frac{n}{2}+p,\ \frac{1}{2}\,(n-2\,p),\ 2,\ 2+\frac{n}{2}+p,\ \frac{a+\frac{1}{x}}{2\,a},\ 1+\frac{1}{a\,x}\right]$$

Result (type 8, 24 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

Problem 934: Unable to integrate problem.

$$\int e^{-2\,p\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

Optimal (type 5, 76 leaves, 3 steps):

$$\frac{1}{a\,(1+2\,p)}\left(1-\frac{1}{a^2\,x^2}\right)^{-p}\left(c-\frac{c}{a^2\,x^2}\right)^{p}\left(1-\frac{1}{a\,x}\right)^{1+2\,p}\text{Hypergeometric2F1}\left[2,\ 1+2\,p,\ 2\,(1+p),\ 1-\frac{1}{a\,x}\right]$$

Result (type 8, 25 leaves):

$$\int e^{-2\,p\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

Problem 935: Unable to integrate problem.

$$\int e^{2\,p\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

Optimal (type 5, 75 leaves, 3 steps):

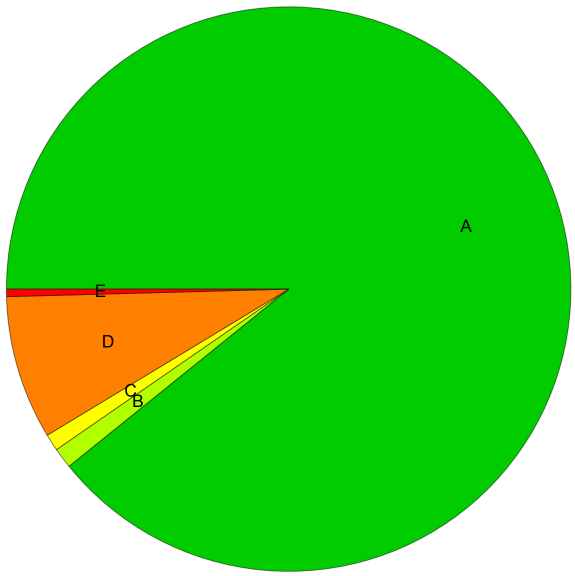
$$-\frac{1}{a\,(1+2\,p)}\left(1-\frac{1}{a^2\,x^2}\right)^{-p}\left(c-\frac{c}{a^2\,x^2}\right)^{p}\left(1+\frac{1}{a\,x}\right)^{1+2\,p}\text{Hypergeometric2F1}\left[2,\ 1+2\,p,\ 2\,(1+p),\ 1+\frac{1}{a\,x}\right]$$

Result (type 8, 25 leaves):

$$\int e^{2\,p\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

Summary of Integration Test Results

935 integration problems



A - 834 optimal antiderivatives

B - 11 more than twice size of optimal antiderivatives

C - 9 unnecessarily complex antiderivatives

D - 77 unable to integrate problems

E - 4 integration timeouts